

# Core Transitions in the Breakup of Exotic Nuclei

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(Dated: February 9, 2008)

An interesting physical process has been unveiled: dynamical core excitation during a breakup reaction of loosely bound *core + N* systems. These reactions are typically used to extract spectroscopic information and/or astrophysical information. A new method, the eXtended Continuum Discretized Coupled Channel (XCDCC) method, was developed to incorporate, in a consistent way and to all orders, core excitation in the bound and scattering states of the projectile, as well as dynamical excitation of the core as it interacts with the target. The model predicts cross sections to specific states of the core. It is applied to the breakup of  $^{11}\text{Be}$  on  $^9\text{Be}$  at 60 MeV/u, and the calculated cross sections are in improved agreement with the data. The distribution of the cross section amongst the various core states is shown to depend on the reaction model used, and not simply on the ground state spectroscopic factors.

PACS numbers: 24.10.Eq, 25.60.Gc, 25.70.De, 27.20.+n

In order to study nuclei at the limit of stability one needs reliable nuclear reactions models that incorporate the relevant structure degrees of freedom in a consistent manner, in particular the continuum. Theories of nuclear reactions have been repeatedly challenged with the new avenue of experimental work now possible at Radioactive Beam Facilities. Amongst the various reactions, breakup occupies a prominent role. With breakup reactions, one tries to extract spectroscopic information [1, 2] or capture reaction rates of Astrophysical relevance [3, 4, 5]. In either case, the structure information obtained is model dependent and assumes a single particle description of the projectile as a valence nucleon attached to the ground state of the core. Such a simplification may be the source of lingering disagreements [6].

When charged particle detectors are coupled with gamma arrays, the states of the core can be disentangled. For the variety of knockout measurements now available [7], the data are found to contain both elastic breakup (diffraction) and transfer (stripping) contributions, which are typically calculated within an eikonal spectator core model [8]. Only the nuclear reaction cross sections to particular states are computed and other effects need to be added *a posteriori*, and incoherently (i.e. [1]). One uses the single particle contributions weighted by the composition of the initial state predicted by shell model [8] and neglects interference between the various projectile components and dynamical processes with the target. However, one should expect that, if the energy necessary to excite the core is small, there will be dynamical core excitation/de-excitation during the reaction with the target. When a loosely bound projectile cannot be described as a single particle state, core excited components are mixed in the projectile scattering states as

well as the ground state. These can interfere during the reaction and modify the single particle picture.

Exotic systems of the type *core + N* where core degrees of freedom may play a relevant role include  $^8\text{Li}$ ,  $^8\text{B}$ ,  $^{11}\text{Be}$ ,  $^{17}\text{C}$ ,  $^{17}\text{F}$  and  $^{19}\text{C}$ . In addition, one can expect to find in their spectra states built on excited states of the core. This is the case of a resonance in  $^{11}\text{Be}$  which is visible at approximately 3.4 MeV excitation in the spectrum of [2], and such states cannot be understood within the single particle description [9]. This calls for a formulation of breakup where core excitation is consistently included.

As a first example, we look at  $^9\text{Be}(^{11}\text{Be}, ^{10}\text{Be})\text{X}$  at 60 MeV/u [1], a reaction already studied in detail. The eikonal model predictions for the total cross sections to particular states are too low:  $\sigma_{\text{th}} = 165 \text{ mb}$  to be compared with  $\sigma_{\text{exp}} = 203(31) \text{ mb}$  for the ground state of  $^{10}\text{Be}$ , and  $\sigma_{\text{th}} = 9 \text{ mb}$  to be compared with  $\sigma_{\text{exp}} = 16(4) \text{ mb}$  for the  $2^+$  excited state. Estimates of a Coulomb contribution and inelastic core excitation presented in [1] provide a possible explanation for the apparent under-prediction of theory, but the problem has been awaiting a consistent formulation. Furthermore, Continuum Discretized Coupled Channel calculations show that the eikonal approximation does not have the desired level of accuracy at this energy[10]. We propose a model where all these corrections and effects are included in a consistent manner.

A recent preliminary study [11] generalizes the spectator core model of [8] to include core excitation. Although the work is performed for nuclear only and within a straight line approximation, it represents an important improvement over previous efforts because a core degree of freedom is introduced consistently in the projectile and the core-target interaction, thus allowing for dynamical core excitation/de-excitation throughout the reaction. The initial results in [11] show an increase of the breakup component of the total cross sections, but no effect on the stripping part. This is an extremely use-

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ful result, that the stripping component seems to be less affected by the various mechanisms discussed above. We will focus here on elastic breakup (diffraction dissociation) only.

As mentioned above, typically breakup models assume a single particle description for the projectile. Only recently are improvements on this approximation being considered. One impressive improvement consists of describing the projectile as a three body system [12] adequate for nuclei of Borromean nature such as  ${}^6\text{He}$ . We pursue an alternative improvement, which is to describe the projectile as a multi-component system based on several core states. In this work, we present the eXtended Continuum Discretized Coupled Channel (XCDCC) method, to take into account explicitly core excitation in the breakup reaction of loosely bound systems.

We consider the breakup of a projectile (P), composed of a core (c) plus a valence particle (v), on a target (T). The breakup process is described using a three body Hamiltonian, with core degrees of freedom denoted by  $\xi$ :  $H^{3b} = T_{\mathbf{R}} + H_{\text{proj}}(\mathbf{r}, \xi) + U_{vt}(\mathbf{r}, \mathbf{R}) + U_{ct}(\mathbf{r}, \mathbf{R}, \xi)$ . The coordinates are illustrated in Fig. 1, and, as Jacobi coordinates, can be used for the full three-body wavefunction

$$\Psi_{J_T M_T}(\mathbf{R}, \mathbf{r}, \xi) = \sum_{\alpha} \Psi_{\alpha}^{J_T}(R) \left[ \left[ Y_L(\hat{\mathbf{R}}) \otimes \Phi_{J_P}^{in}(\mathbf{r}, \xi) \right]_J \otimes \Phi_{J_t} \right]^{J_T M_T}, \quad (1)$$

where  $L$  is the projectile-target orbital angular momentum,  $J_t$  the total spin of the target,  $J_T$  the total spin of the three-body system, and  $\alpha = \{L, J_P, J, J_t, i, n\}$ .

The projectile states  $\Phi_{J_P}(\mathbf{r}, \xi)$  can be either a bound or a scattering state, with several components. They are obtained as coupled channels eigenstates of the projectile Hamiltonian  $H_{\text{proj}} = T_{\mathbf{r}} + V_{vc}(\mathbf{r}, \xi) + h_{\text{core}}(\xi)$ :

$$\Phi_{J_P}^{in}(\mathbf{r}, \xi) = \sum_a u_{a:n}^i(r) [[Y_l(\hat{\mathbf{r}}) \otimes \chi_s]_j \otimes \varphi_I(\xi)]_{J_P}, \quad (2)$$

expanded in terms of core eigenstates at energies  $\varepsilon_I$ .

A continuum projectile state is characterised by  $\{J_P, i, n\}$ , where  $i$  refers to the asymptotic energy, and  $n$  denotes the channel with a plane wave component. It is composed of projectile radial wavefunctions  $f_{a:n}(r; k_{an})$  for each  $a = \{lsj, I\}$  that are solutions of coupled equa-

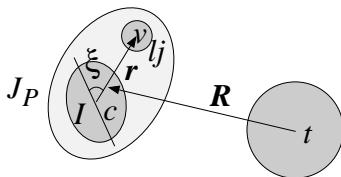


FIG. 1: Coordinates for the three body breakup reaction.

tions which, since the state of the core can change, are

$$\begin{aligned} \left[ E_k - \varepsilon_a + \varepsilon_n + \frac{\hbar^2}{2\mu_{vc}} \left( \frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} \right) \right] f_{a:n}(r; k_{an}) \\ = \sum_{a'} V_{a:a'}(r) f_{a':n}(r; k_{a'n}), \quad (3) \end{aligned}$$

where the coupling potentials are matrix elements of  $V_{vc}(\mathbf{r}, \xi)$ . From these equations we obtain the S-matrix  $S_{a:n}$ .

Given the importance of continuum-continuum couplings in the CDCC formulation of breakup for halo nuclei [13], we need to transform the projectile scattering states into square integrable functions, otherwise continuum-continuum couplings would diverge. We define a coupled channel bin as

$$u_{a:n}^i(r) = \sqrt{\frac{2}{\pi(k_i - k_{i-1})}} \int_{k_{i-1}}^{k_i} dk e^{-i\delta_n(k)} f_{a:n}(r; k_{an}) \quad (4)$$

where  $k$  is the core-valence relative momentum,  $k_{an}^2 = k^2 - \frac{2\mu_{vc}}{\hbar^2} [\varepsilon_a - \varepsilon_n]$ . From  $S_{a:n}$  we obtain  $\delta_n(k)$ , the diagonal valence-core phase shift of channel  $a = n$ . Coupled channel bins defined in this way are complex.

Substituting the three body wavefunction Eq. (1) into the three body Schrödinger equation  $H^{3b}\Psi_{J_T} = E\Psi_{J_T}$ , one obtains an equation set similar to the standard CDCC equations of [14], with coupling potentials  $U_{\alpha:\alpha'}^{J_T}(R) = \langle \alpha | U_{ct}(\mathbf{r}, \mathbf{R}, \xi) + U_{vt}(\mathbf{r}, \mathbf{R}) | \alpha' \rangle$  containing both Coulomb and nuclear interactions between the projectile fragments and the target. The techniques for solving this equation are the same as in [13]. However there are essentially two differences in these eXtended CDCC equations: first they span a larger number of projectile coupled components, corresponding to core excitation, and second the interaction core-target depends on core degrees of freedom  $\xi$ . In our calculations, the couplings  $U_{\alpha:\alpha'}^{J_T}(R)$  are further expanded in multipoles and non-trivial algebra is necessary to simplify the problem, but we leave the details on the evaluations of these matrix elements for a longer publication [15]. A new generation of the code FRESCO [16] was developed to incorporate these aspects, namely coupled channel bins and core excitation CDCC couplings. The code was further optimized and parallelized so that realistic calculations could become feasible.

The first application of XCDCC is to the breakup components of  ${}^9\text{Be}({}^{11}\text{Be}, {}^{10}\text{Be})\text{X}$  at 60 MeV/u [1]. As the spin  $s$  of the neutron has an insignificant effect, we set it to  $s = 0$ . We include only the first excited state of the core as in [18]. For our case the ground state of  ${}^{11}\text{Be}$  contains two components: an s-wave neutron coupled to  ${}^{10}\text{Be}(0^+)$  and a d-wave neutron coupled to  ${}^{10}\text{Be}(2^+)$ . The model for  $n-{}^{10}\text{Be}$  is based on [18], but the depths of the Woods-Saxon interactions need to be adjusted to give a positive parity bound state energy at  $E_+ = -0.5$  MeV and a negative parity level at  $E_- = -0.18$  MeV, in the  $s = 0$

model	interaction	$V(+)$	$V(-)$	$R$	$a$	$\beta_2$
CC	$^{10}\text{Be}\{0^+, 2^+\} + \text{n}$	55.25	47.00	2.483	0.65	0.67
SP	$^{10}\text{Be}\{0^+\} + \text{n}$	55.50	30.48	2.736	0.67	0.0
SP	$^{10}\text{Be}\{2^+\} + \text{n}$	75.07	39.95	2.736	0.67	0.0
model	interaction	$V$	$W$	$r$	$a$	$\beta_2$
CC	$^{10}\text{Be}-^{9}\text{Be}$	134.1	68.15	0.75	0.8	0.67

TABLE I: Potential parameters for  $^{10}\text{Be} + \text{n}$  with and without deformation of the core. Also given are the parameters for the core-target optical potential.

approximation. The needed interaction in the coupled channel model (CC) of  $^{11}\text{Be}$  is listed in the first row of table I. It produces a ground state of  $^{11}\text{Be}$  where the neutron is 88.3% in the s-wave and 11.7% in the d-wave. Note that these probabilities can be related to the spectroscopic factors in the shell model which are normalized to particle number [17]. For comparison we also want to calculate the breakup cross section using the previous CDCC approach. In this calculation, each single particle contribution is multiplied by the corresponding projectile's probability and all components are added incoherently. Therefore we label this calculations by SPIS for single particle incoherent sum). SPIS neglects core excitation during breakup, and uses the single particle potentials built on the g.s. of the core and its first excited state as listed in the second and third rows of table I. These potentials are used to generate the bound states and the whole continuum (these structure calculations are referred to as SP for single particle).

The neutron-target optical potential parameters are taken from [19]. As the target  $^{9}\text{Be}$  is a spectator, its spin  $J_t$  is neglected, as in Refs [1, 7, 8, 10, 11]. For the  $^{10}\text{Be}-^{9}\text{Be}$  we take the potentials from [20] that reproduce unpublished data for the elastic scattering of  $^{10}\text{Be}$  on  $^{12}\text{C}$  at 59 MeV/u. This potential is directly used in the SP calculations. For the XCDCC calculations, we deformed both the nuclear and the Coulomb parts using the deformation lengths consistent with our structure model for  $^{11}\text{Be}$  [21] and refit the potential to reproduce the same elastic distribution. The resulting potential is given in the last row of table I.

We briefly describe the model space for solving the XCDCC equations. We take partial waves for  $J_P \leq 4$  organized in 178 bins as schematically shown in Fig. 2. The evaluation of the couplings  $U_{\alpha:\alpha'}^{J_T}(R)$  involve an integration in  $r$  which is performed up to  $r_{\text{bin}} = 70$  fm, and a multipole expansion which we truncate at  $\Lambda_{\text{max}} = 4$ . The projectile-target relative angular momentum is taken up to  $L_{\text{max}} = 3000$  and the corresponding distorted waves are matched to Coulomb functions at  $R_{\text{asym}} = 500$  fm. The XCDCC calculations were performed on a SGI Altix 3700.

The results are shown in table II. Using a single particle model for  $^{11}\text{Be}$ , and introducing ground state occupation probabilities consistent with the ones produced by the CC model, namely 88.3 % s-wave and 11.7 % d-wave, the predicted cross sections are 109 mb to the g.s.

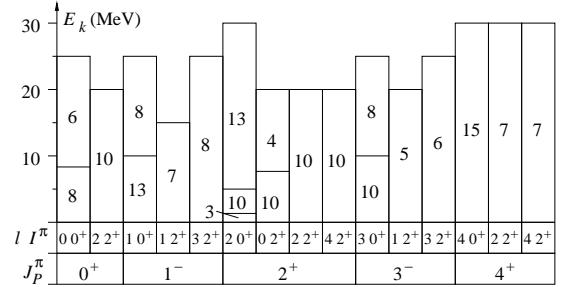


FIG. 2:  $^{11}\text{Be}$  continuum model space. The number of bins and the energy range are given for each outgoing channel ( $l, I^\pi$ ) for each spin parity combination of the projectile ( $J_P^\pi$ ).

Model	$\sigma_{0+}$	$\sigma_{2+}$	$\sigma$
Eikonal	105 mb	3.4 mb	108 mb
SPIS	109 mb	1 mb	110 mb
XCDCC	109 mb	8 mb	117 mb

TABLE II:  $^{11}\text{Be}$  breakup cross sections for  $^{10}\text{Be}\{0^+, 2^+\} + \text{n}$ .

of the core, and 1 mb to the first excited state (model SPIS). In comparison, when core excitation is included consistently, the XCDCC calculations predict exactly the same cross section to the g.s. of the core, but a large increase (a factor of 8) in the cross section to the  $^{10}\text{Be} 2^+$  state. In both SPIS and XCDCC breakup, the probabilities for seeing the  $2^+$  state,  $P_2 = \sigma_2/\sigma_{\text{tot}}$ , are much lower than in the ground state. This reflects the fact that the partial cross section decreases rapidly with increasing single-particle Q-value for breakup.

The occupation probability (or for that matter the spectroscopic factor) is not an observable, therefore it is not good practice to compare theory and experiment at this level. Rather one should include an appropriate reaction model and compare cross sections. If we take the eikonal prediction for nuclear breakup directly from [1] and include occupation probabilities consistent with our  $^{11}\text{Be}$  coupled channel model, we obtain cross sections lower than the XCDCC predictions (see first row of table II). The differences between the theoretical cross sections predicted in the Eikonal model and the data [1] were attributed to the Coulomb breakup and inelastic excitation of the core. In XCDCC one can turn off the various couplings to understand their relative importance. We find that even when core excitation couplings are not included in the core-target interaction, the cross section to the  $2^+$  is increased over the SPIS prediction, due to constructive interference between the projectile's components in the continuum. There are also nuclear-Coulomb interference effects. Consequently, inelastic and Coulomb contributions should not be added incoherently. These are automatically contained in the XCDCC predictions.

For a meaningful comparison with the data one needs to construct the stripping component, since the neutron was not detected in the measurement [1]. However XCDCC produces the elastic breakup component only.

core state	$\sigma_{bu}$	$\sigma_{st}$	$\sigma_{th}$	$\sigma_{exp}$
$0^+$	109 mb	91 mb	200	203(31)
$2^+$	8 mb	6 mb	14	16(4)

TABLE III: Comparison of calculated cross sections, sum of XCDCC breakup and Eikonal stripping, with the data [1] for  ${}^9\text{Be}({}^{11}\text{Be}, {}^{10}\text{Be})\text{X}$  at 60 MeV/u.

In principle we would like to produce the stripping in the same framework as the breakup but this is at present not possible. From the work of Batham *et al.* [11] we learned that the stripping is hardly affected by core excitation, so we take the stripping contribution calculated in the eikonal approximation [22], based on the same optical potentials as the XCDCC calculation here presented. The results are summarized in table III and immediately one can see that the theoretical cross sections agree perfectly with the data.

Since the core-target optical potential was scaled from that obtained from elastic scattering of  ${}^{10}\text{Be}$  on a similar target ( ${}^{12}\text{C}$ ), we test the sensitivity on the choice of this potential. For this purpose, we use a microscopically based potential, by folding the NN interaction over the density of the core, to reproduce the same elastic and inelastic cross sections for  ${}^{10}\text{Be}$  as the potential of table I. The XCDCC breakup cross sections to each core state are not affected, and neither are the corresponding stripping cross sections. This demonstrates that details of the core-target optical potential are not important, as long as similar observables for the core-target interaction are obtained.

Finally, it is necessary to point out that within our model space, there is no population of the  ${}^{10}\text{Be}$   $1^-$  and  $2^-$  states, in this reaction. These states are seen in the experiment [1], but we expect that these states, which result from neutron excitation from the core, will not interfere with the results here presented. At present we do not have a  ${}^{10}\text{Be}$  structure model that incorporates all  $2^+, 1^-, 2^-$  states in a simple way, as these correspond to breaking the core.

In conclusion, we find that the amount of core excitation is modified in the breakup reaction. This process occurs both through constructive interference of various components in the projectile and through the interac-

tion of the core with the target. The eXtended Continuum Discretized Coupled Channel method was developed specifically to handle the problem of core excitation in the breakup of loosely bound projectiles. Effects of core excitation in the projectile bound and scattering states are explicitly taken into account. Through the interaction with the target, the core can excite or de-excite during the reaction. We have applied XCDCC to the breakup of  ${}^{11}\text{Be}$  on  ${}^9\text{Be}$  at 60 MeV/u, where the final  ${}^{10}\text{Be}$  state is identified. Theoretical predictions within a truncated model space agree with the data for the cross sections to the first two individual  ${}^{10}\text{Be}$  states.

As compared to the preliminary calculations of Batham *et al.* [11], XCDCC provides more detailed cross sections, namely partial cross sections to each core state. As a consequence we now understand that the increase of the total cross section when including core excitation comes mainly from an increase of the core excited component. Thus one can think of it as production of core excitation. This process was not understood before.

Despite the computational challenge, XCDCC consists on a significant improvement to previous theories. Given the possibility of producing core excitation, previous spectroscopic analyses and extractions of astrophysical S-factors need to be revisited. Other reactions that can be usefully studied with XCDCC include  ${}^{11}\text{Be}(p,p'){}^{11}\text{Be}^*$ ,  ${}^{12}\text{C}({}^{11}\text{Be}, {}^{10}\text{Be}+n)$ , the various modes of  ${}^8\text{B}$  breakup, and work along these lines is underway. So far, the method is limited to the inclusion of states of the core that can be modelled as collective excitations, but it could easily be adapted to including a better description of the core, as long as a complete set of  $\text{core} + N$  (bound and scattering states) could be obtained.

We thank Jeff Tostevin for providing the stripping cross sections for comparison with the data and for useful discussions. We also thank Jim Al-Khalili for providing the data for  ${}^{10}\text{Be}$ . This work is supported by NSCL, Michigan State University, the U.K. EPSRC by grant GR/T28577, and by the National Science Foundation through grant PHY-0456656. The calculations presented were performed at the HPCC at Michigan State University.

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